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NECESSARY AND SUFFICIENT FRACTURE CRITERIA FOR A COMPOSITE WITH A BRITTLE MATRIX. PART 1. WEAK REINFORCEMENT

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Fracture of a composite medium with a brittle matrix is studied. The brittle or plastic material of the reinforcing elements is highly deformable. For normal-rupture macrocracks, necessary criteria of brittle strength and sufficient criteria of quasibrittle strength are proposed. Simple analytical dependences of the macrocrack length on the loading parameter, structural, rigidity, and strength parameters of the medium, and damage parameters of the material of the components are obtained. The critical loads for these criteria may differ substantially even if the reinforcement coefficient is small and the material of reinforcing elements is highly deformable. If the necessary criterion is satisfied, crack extension occurs and microcracks are formed in the bonds of the structure located ahead of the macrocrack tip. The number of damaged bonds depends on the macrocrack opening and characteristics of postcritical deformation of the damaged bonds.

Introduction. Neuber [1] and Novozhilov [2] studied fracture of brittle bodies with various quasiregular structures. In the Neuber–Novozhilov criteria [1, 2], the averaging strongly depends on the characteristic linear dimensions of a regular-structure elementary cell. A composite material is a typical structured medium. In studying the composite fracture, one encounters problems that differ considerably from the classical problems of the elasticity theory. Below, we give an example where the regular structure of a composite is determined by two characteristic linear sizes: the diameter of highly deformable inclusions and the distance between these inclusions. The author [3, 4] considered the fracture of brittle isotropic solids with a hierarchy of regular structures, each being described by only one linear size.

We first briefly review the necessary and sufficient fracture criteria for isotropic solids with one structure. The necessary criteria considered in [2, 4] can be used to describe fracture of composites with a brittle matrix. The use of the sufficient criterion [2] proposed for homogeneous bodies can lead to erroneous results in calculating composites. Let the necessary criterion be satisfied. Then the structure of the body nearest to the crack tip is in the critical state. However, after the critical load of this structure is exceeded, the cracked body can sustain additional loading owing to simultaneous postcritical deformation of the structure ahead of the crack tip and subcritical deformation of the next structure. If the sufficient criterion is satisfied, the critical load of simultaneous deformation of two or more structures ahead of the crack tip is exceeded, and the body with a crack is broken into fragments.

Let us consider the classical sufficient criteria [5–8] in more detail. Nazarov and Polyakova [9] gave a mathematical interpretation of the Leonov–Panasyuk–Dugdale model [5, 6]. For the two-term asymptotic representation of the solution (isotropic material), the stresses on the continuation of the crack ($y = 0$) in the vicinity of its tip can be written with accuracy to higher-order terms as

$$\sigma_y(x, 0) \simeq \sigma_\infty + K_{\rm I}^0/(2\pi x)^{1/2}, \tag{1}$$

where σ_∞ is the characteristic stress at infinity and $K_{\rm I}^0$ is the total stress-intensity factor (SIF). The total SIF can be written in the form [10]

$$K_{\rm I}^0 = K_{{\rm I}\infty}^0 + K_{{\rm I}\Delta}^0, \quad K_{{\rm I}\infty}^0 > 0, \quad K_{{\rm I}\Delta}^0 < 0. \tag{2}$$

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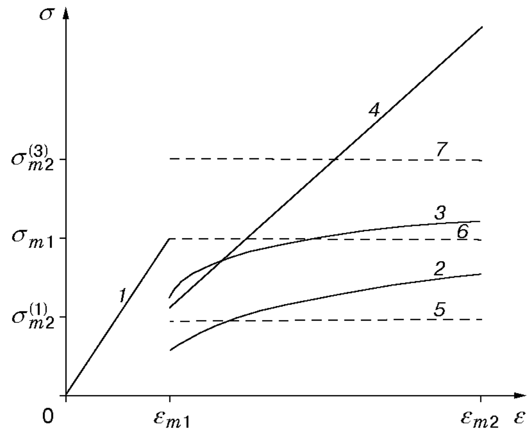

Fig. 1

Here $K^0_{\mathrm{I}\infty}$ is the SIF induced by the stresses σ_∞ and $K^0_{\mathrm{I}\Delta}$ is the SIF induced by the stresses σ_m that act, in accordance with the Leonov–Panasyuk–Dugdale model, in the vicinity of the crack tip in the pre-fracture zone (Δ is the length of the loaded region or pre-fracture zone). The stresses σ_m coincide with the "theoretical" strength of a single crystal [2, 10]. The restrictions

$$K^0_{\mathrm{I}} = 0, \tag{3}$$

$$K^0_{\mathrm{I}} > 0 \tag{4}$$

are imposed on relations (1) and (2), respectively. In the classical criteria [6, 7], restriction (3) is essentially used. The class of solutions corresponding to restriction (4) is analyzed in [10, 11].

Remark 1. The total SIF K^0_{I} cannot be negative since, for $K^0_{\mathrm{I}} < 0$, the crack flanks overlap with one another, which can be easily verified.

Below, we consider restriction (4), which implies that relation (1) contains a singular component of the solution. Infinite stresses at the crack tip, which are inconsistent with the continual criterion, do not contradict the discrete criterion [1, 2] if the singular component has an integrable singularity.

For analysis, the pre-fracture zone adjacent to the crack tip is of primary interest. We denote the quantities that refer to the initial composite and reinforcing fibers (inclusions) by the subscripts 1 and 2, respectively. For the model of a composite material in question, the stresses $\sigma_{m2} = \text{const}$ in the pre-fracture zone may differ from the "theoretical" strength σ_{m1} of a bundle of fibers with a brittle matrix. The following cases are possible: 1) $\sigma_{m1} > \sigma_{m2}$; 2) $\sigma_{m1} = \sigma_{m2}$; 3) $\sigma_{m1} < \sigma_{m2}$. The second case with restriction (4) was considered in [10, 11]. The first case corresponds to weak reinforcement and the third case to high-strength reinforcing fibers; in the first and second cases, restrictions (3) or (4) can be satisfied.

It should be noted that the use of the necessary criterion of brittle strength and sufficient criterion of quasibrittle strength proposed by Novozhilov [2] seems to be reasonable in the analysis of the pre-fracture and final rupture of a composite.

1. Physicomechanical Model of a Bundle of Fibers for the Pre-Fracture Zone. Let the initial composite have a regular structure characterized by one geometrical parameter r_1 (r_1 is the distance between the fibers-inclusions). For simplicity, we assume that, after averaging, the composite is described by the equations of isotropic elastic media everywhere outside the pre-fracture zone. In the pre-fracture zone, the behavior of the partly ruptured composite depends on its structure and the σ–ε relation of reinforcing fibers. The simplest model of a composite is the fiber-bundle model. It should be noted that the structure of the composite can contain no fibers before fracture. Below, we consider an example where fibers are formed from inclusions in the pre-fracture zone upon crack propagation only through the brittle matrix. The material of these inclusions is assumed to be highly deformable.

Let each representative bundle of fibers be bonded by a brittle matrix and reinforcing fibers be brittle or plastic. We assume that the limiting relative elongation of the fibers is much greater than that of the matrix. Figure 1 shows simplified σ–ε diagrams of a fiber bundle and their approximations. The solid straight line 1 refers to elastic deformation of the composite (fiber bundle), and the solid curves 2–4 refer to the first ($\sigma_{m1} > \sigma_{m2}^{(1)}$), second ($\sigma_{m1} = \sigma_{m2}^{(2)}$), and third ($\sigma_{m1} < \sigma_{m2}^{(3)}$) cases of fiber deformation where the matrix is disrupted. The dashed straight lines 5–7 are approximations of curves 2–4, respectively. In Fig. 1, the following notation is used: σ_{m1} is the "theoretical" strength of the fiber bundle with allowance for the damaged matrix, ε_{m1} and ε_{m2} are the limiting relative elongations of the matrix and fibers ($\varepsilon_{m1} < \varepsilon_{m2}$), $\sigma_{m2}^{(1)}$, $\sigma_{m2}^{(2)}$, and $\sigma_{m2}^{(3)}$ are the averaged stresses acting in the pre-fracture zone for the first, second, and third cases, respectively ($\sigma_{m2}^{(2)} = \sigma_{m1}$). In the Leonov–Panasyuk–Dugdale model, these averaged stresses are calculated using energy considerations [2, 5, 6, 10, 11]:

$$\sigma_{m2} = \frac{1}{\varepsilon_{m2} - \varepsilon_{m1}} \int\limits_{\varepsilon_{m1}}^{\varepsilon_{m2}} \sigma(\varepsilon)\, d\varepsilon. \tag{5}$$

Figure 1 and relation (5) refer to "testing" of a fiber bundle of the composite in an extremely rigid machine.

The phenomenological models in fracture mechanics are described in [12, 13]. The mechanism of fracture of a composite with a brittle matrix is the growth of a macrocrack in the matrix followed by breakage of fibers [12]; only after fibers are broken, the composite disintegrates.

In formulating the strength criteria, it is impossible to use the quantities ε_{m1} and ε_{m2} directly since the characteristic linear size is absent. We consider the cross-sectional size of the pre-fracture zone and denote it by a (for a single crystal, this size is equal to the lattice constant [2, 10]; in Sec. 3, the linear size a is the diameter of inclusions). Thus, the pre-fracture zone occupies a rectangle with the sides Δ and a (see [14]). The crack opening at the point $x = -\Delta$ is determined as $a_{m2} = (\varepsilon_{m2} - \varepsilon_{m1})a$. In the pre-fracture zone, the constant stresses σ_{m2} act in the interval $[-\Delta, 0)$ (the origin of the coordinate system is located at the right tip of the crack). The stresses σ_{m2} "try" to close the crack.

Finally, we have two sets of parameters: the geometric parameters r_1, Δ, and a_{m2} and the force parameters σ_{m1} and σ_{m2}. The first parameters of these sets, r_1 and σ_{m1}, are used to formulate the necessary criterion of brittle strength, and the complete set of parameters is used to formulate the sufficient criterion of quasibrittle strength.

2. Necessary Criterion of Brittle Strength and Sufficient Criterion of Quasibrittle Strength of a Composite. A normal-rupture crack in a composite is modeled by a bilateral cut. The necessary discrete–integral criterion of brittle strength has the form ($\Delta = 0$)

$$\frac{1}{kr_1} \int\limits_0^{nr_1} \sigma_y(x, 0)\, dx \leqslant \sigma_{m1}, \qquad x \geqslant 0. \tag{6}$$

The sufficient discrete–integral criterion of quasibrittle strength has the following form ($\Delta > 0$ and $a_{m2} > 0$):

$$\frac{1}{kr_1} \int\limits_0^{nr_1} \sigma_y(x, 0)\, dx \leqslant \sigma_{m1}, \quad x \geqslant 0; \qquad 2v^* = \frac{\text{æ} + 1}{G}\, K_{\mathrm{I}}^0 \sqrt{\frac{\Delta}{2\pi}} \leqslant a_{m2}, \quad x \leqslant 0. \tag{7}$$

Here σ_y are the normal stresses on the continuation of the crack (they can have a singular component), Oxy is the Cartesian coordinate system whose origin coincides with the right tip of the crack, r_1 is the characteristic linear size of the composite-material structure, n and k are numbers ($n \geqslant k$, where k is the number of undamaged fibers), nr_1 is the interval of averaging, $(n - k)/n$ is the coefficient that takes into account damaged reinforcement in this interval, $2v = 2v(x)$ is the crack opening, $2v^*(-\Delta) = a_{m2}$ is the critical crack opening for which the fiber nearest to the crack center fails, $\text{æ} = 3 - 4\nu$ or $\text{æ} = (3 - \nu)/(1 + \nu)$ for plane strain and plane stress, respectively (ν is Poisson's ratio), G is the shear modulus, and K_{I}^0 is the total SIF calculated from relation (2) for the corresponding problem. The stresses in criteria (6) and (7) are averaged within the limits that depend on the presence and location of defects (failure of fibers) in the pre-fracture zone of the deformed composite. Damage in the pre-fracture zone strongly depends on the initial damages of the composite.

3. Crack Extension in a Composite upon Formation of Force Bonds at the Crack Tip. There are brittle materials (for example, ceramics) that easily fail under extension in the presence of crack-like macrodefects. Introducing small amounts of highly deformable additives (weak reinforcement) significantly changes the behavior of the composite in the pre-fracture zone. Problems related to crack propagation in media with such a structure were considered in [15, 16]. The following highly deformable components were introduced into a brittle ceramic matrix: gold (metal) or Teflon (solid polymer). It was found that, under certain conditions, these materials possess

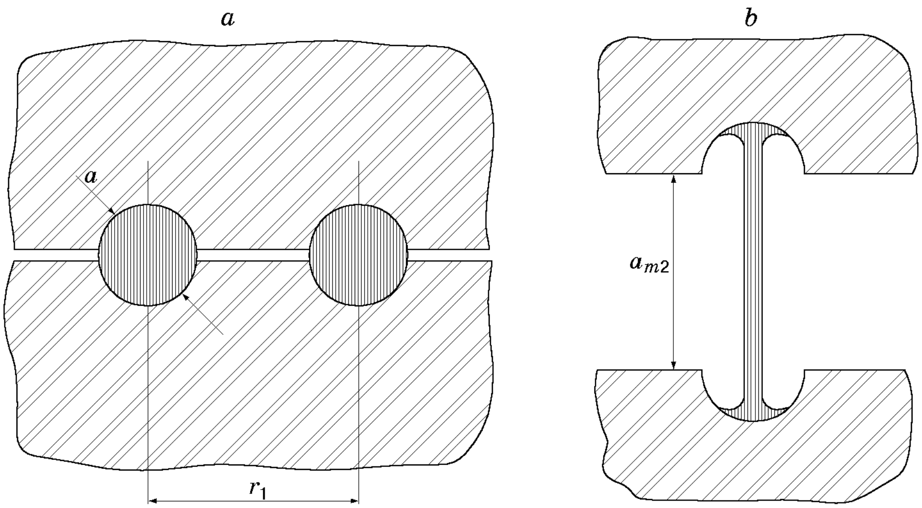

Fig. 2

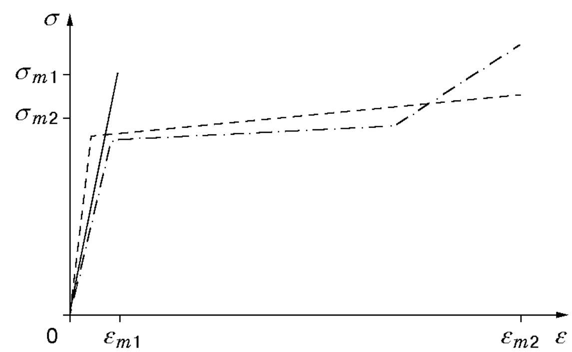

Fig. 3

a higher crack resistance. We use criteria (6) and (7) to interpret the effects observed in [15, 16]. We consider a two-dimensional formulation of the problem.

3.1. *Stress State in the Vicinity of the Crack Tip.* We study the quasistatic extension of plane cracks in a composite with a highly deformable component. Let this component form a system of sparse inclusions in a brittle ceramic matrix (Fig. 2a). It is assumed that cylindrical inclusions are located regularly (a is the diameter of the inclusions and r_1 is the distance between their centers). A plane crack passes through the centers of the inclusions, and the forces acting on the ceramics–metal or ceramics–polymer interfaces do not exceed the adhesion forces between ceramics and metal (for example, gold) or ceramics and solid polymer (for example, Teflon). In this case, it may be assumed that force bonds are formed from the cylindrical inclusions (Fig. 2b). The maximum crack opening can be estimated as $a_{m2} \simeq a(\varepsilon_{m2} - \varepsilon_{m1})$, where ε_{m1} and ε_{m2} are the maximum strains in the matrix and bonds and $\varepsilon_{m1} \ll \varepsilon_{m2}$ (Fig. 3). Figure 3 shows the σ–ε diagrams, where σ_{m1} and σ_{m2} are the maximum stresses in the matrix and bonds, respectively. We note that the stresses σ_1 and σ_2 in the solid with a structure have already been averaged. In Fig. 3, the dashed and dot-and-dashed curves are typical σ–ε diagrams for highly deformable metals and solid polymers, respectively, and the solid line is the σ–ε diagram for brittle ceramics.

We consider two models of a crack. In the first model, bonds are absent inside the crack (necessary criterion), whereas in the second model, force bonds exist in the vicinity of the crack tip (sufficient criterion).

We study the behavior of a composite in the vicinity of the tip of a rupture crack. It is assumed that force bonds are absent inside the crack in the initial state and the stress σ_∞ is specified at infinity. Let the stress σ_∞ be lower than the critical stress σ_∞^0 for the necessary criterion. In this case, the crack does not grow since $\sigma_\infty < \sigma_\infty^0$. Under further gradual loading, the stresses σ_∞ attain the critical value σ_∞^0. For $\sigma_\infty = \sigma_\infty^0$, the crack begins to grow because of failure of the matrix, force bonds are formed from the more deformable material in the vicinity of the

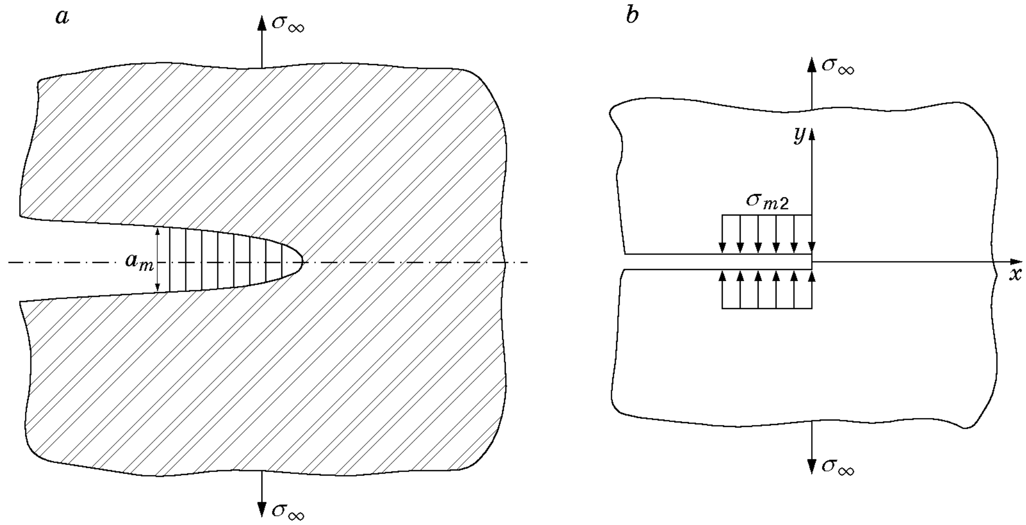

Fig. 4

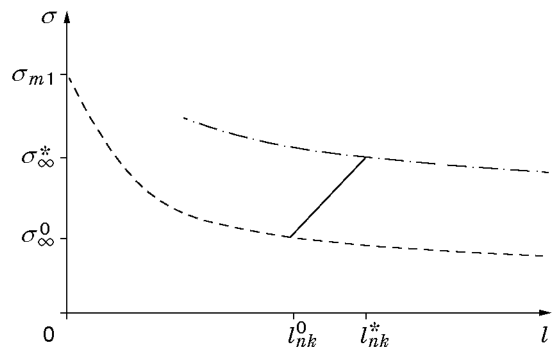

Fig. 5

crack tip, and a pre-fracture zone appears (Fig. 4a). In accordance with the Leonov–Panasyuk–Dugdale model, the stresses σ_{m2} at the loaded part of the crack "try" to close the crack (Fig. 4b). The force bond nearest to the middle of the crack fails when its relative elongation reaches the critical value ε_{m2}. The difference between the mechanical models considered is as follows: a loaded part ($\Delta = 0$) is absent for the necessary criterion, whereas a loaded part ($\Delta > 0$) exists for the sufficient criterion. Obviously, one can pass from one mechanical model to the other in the limit as $\Delta \to 0$. In the postcritical region, the "reinforcement" allows the composite to sustain the load additional to σ_∞^0.

3.2. *Comparison of the Critical Stresses Predicted by the Necessary and Sufficient Criteria.* According to the models chosen, a pre-fracture zone is absent in the necessary criterion, whereas this zone exists in the sufficient criterion. For these criteria, we express the stresses on the continuation of sharp cracks ($y = 0$) in terms of the total SIF in the form [17]

$$\sigma_y(x,0) = \sigma_\infty^0 + K_{\rm I}^0/(2\pi x)^{1/2} \qquad \Big(K_{\rm I}^0 = \sigma_\infty^0 \sqrt{\pi l_{nk}^0}, \qquad \Delta = 0\Big), \tag{8}$$

$$\sigma_y(x,0) = \sigma_\infty^* + K_{\rm I}^*/(2\pi x)^{1/2}. \tag{9}$$

Here $K_{\rm I}^* = \sigma_\infty^* \sqrt{\pi l_{nk}^*} - \sigma_{m2}\sqrt{\pi l_{nk}^*}[1 - (2/\pi)\arcsin(1 - \Delta/l_{nk}^*)]$, $\Delta \neq 0$, $2l_{nk}^0 = 2l^0(n,k)$ and $2l_{nk}^* = 2l^*(n,k)$ are the critical lengths of the cracks, $K_{\rm I}^0$ and $K_{\rm I}^*$ are the critical SIFs, and σ_∞^0 and σ_∞^* are the critical stresses for the necessary and sufficient criteria, respectively.

Let there be a crack of length $2l_{nk}^0$ such that $\Delta = 0$. Figure 5 shows the fracture curves. The dashed curve refers to the necessary criterion, the dot-and-dashed curve refers to the sufficient criterion, and the solid line shows the passage of the system from one state of equilibrium (pre-fracture zone is absent) to the other (pre-fracture zone exists). The dashed and dot-and-dashed curves refer to unstable crack growth, and the solid curve refers to stable crack growth. During stable growth, the crack extension occurs ($l^0_{nk} < l^*_{nk}$) and force bonds are formed at the crack tip (Fig. 4). The new system can sustain an increasing load since $\sigma^*_\infty > \sigma^0_\infty$. A peculiar trap for propagating cracks is formed [15, 16].

We obtain relations between the critical parameters for the two criteria proposed. One can easily estimate the critical SIFs K^0_{I} and K^*_{I}. After some transformations, we have

$$\frac{K^0_{\mathrm{I}}}{\sigma^0_\infty\sqrt{r_1}} = \sqrt{\frac{\pi}{2n}}\Big(k\,\frac{\sigma_{m1}}{\sigma^0_\infty} - n\Big), \qquad \frac{K^*_{\mathrm{I}}}{\sigma^*_\infty\sqrt{r_1}} = \sqrt{\frac{\pi}{2n}}\Big(k\,\frac{\sigma_{m1}}{\sigma^*_\infty} - n\Big). \tag{10}$$

In contrast to the first relation in (10), which can be used directly, the second relation contains an unknown parameter Δ characterizing the length of the pre-fracture zone [see (9)]. Relations (9) and (10) make sense if $K^*_{\mathrm{I}} > 0$. It is noteworthy that criteria (6) and (7) proposed make sense also for a zero length of the crack since the terms σ^0_∞ and σ^*_∞ are retained in relations (8) and (9). Thus, the first relation in (10) can be used to describe crack initiation.

The last relation in (9) is simplified substantially if Δ/l^*_{nk} is a small quantity. In this case, we obtain

$$\arcsin\,(1 - \Delta/l^*_{nk}) \simeq \pi/2 - \sqrt{2\Delta/l^*_{nk}}.$$

With allowance for this simplification, relations (7) and (9) yield the quadratic equation for the parameter $\sqrt{\Delta/l^*_{nk}}$:

$$\bigg(\sqrt{\frac{\Delta}{l^*_{nk}}}\bigg)^2 - \frac{\pi}{2\sqrt{2}}\,\frac{\sigma^*_\infty}{\sigma_{m2}}\sqrt{\frac{\Delta}{l^*_{nk}}} + \frac{\pi}{æ+1}\,\frac{a\varepsilon_{m2}}{l^*_{nk}}\,\frac{G}{2\sigma_{m2}} = 0.$$

We use the following simplification to calculate the roots of the last equation. Let

$$\frac{8}{\pi(æ+1)}\,\frac{a\varepsilon_{m2}}{l^*_{nk}}\,\frac{\sigma_{m2}}{\sigma^*_\infty}\,\frac{G}{\sigma^*_\infty} \ll 1.$$

Ignoring the squared quantity compared to unity, we obtain the explicit expression for the smaller root of the quadratic equation:

$$\sqrt{\frac{\Delta}{l^*_{nk}}} = \frac{\sqrt{2}}{æ+1}\,\frac{a\varepsilon_{m2}}{l^*_{nk}}\,\frac{G}{\sigma^*_\infty}.$$

Finally, the critical length of a rupture crack is written in the explicit form:

— for the necessary criterion ($\Delta = 0$),

$$\frac{2l^0_{nk}}{r_1} = \Big(\frac{\sigma_{m1}}{\sigma^0_\infty} - \frac{n}{k}\Big)^2\frac{k^2}{n}, \qquad \frac{\sigma_{m1}}{\sigma^0_\infty} = \frac{\sqrt{n}}{k}\sqrt{\frac{2l^0_{nk}}{r_1}} + \frac{n}{k}; \tag{11}$$

— for the sufficient criterion ($\Delta \neq 0$),

$$\frac{2l^*_{nk}}{r_1} = \Big(\frac{\sigma_{m1}}{\sigma^*_\infty} - \frac{n}{k}\Big)^2\frac{k^2}{n}\bigg(1 - \frac{2\sigma_{m2}}{\pi\sigma^*_\infty}\sqrt{\frac{\Delta}{l^*_{nk}}}\bigg)^{-2}, \qquad \sqrt{\frac{\Delta}{l^*_{nk}}} = \frac{\sqrt{2}}{æ+1}\,\frac{a\varepsilon_{m2}}{l^*_{nk}}\,\frac{G}{\sigma^*_\infty},$$
$$\frac{\sigma_{m1}}{\sigma^*_\infty} = \frac{\sqrt{n}}{k}\sqrt{\frac{2l^*_{nk}}{r_1}}\bigg(1 - \frac{2\sigma_{m2}}{\pi\sigma^*_\infty}\sqrt{\frac{\Delta}{l^*_{nk}}}\bigg) + \frac{n}{k}. \tag{12}$$

The critical parameters K^0_{I}, K^*_{I}, l^0_{nk}, and l^*_{nk} in relations (10)–(12) admit the limiting passage when the SIFs and lengths of the cracks vanish (in classical relations, a similar limiting passage does not make sense).

The critical lengths of the cracks l^0_{nk} and l^*_{nk} differ for the same value of σ_∞. This difference can be rather significant:

$$\frac{l^*_{nk}}{l^0_{nk}} = \bigg(1 - \frac{2\sqrt{2}}{\pi(æ+1)}\,\frac{\sigma^{(2)}_m}{\sigma_\infty}\,\frac{a\varepsilon_{m2}}{l^*_{nk}}\,\frac{G}{\sigma_\infty}\bigg)^{-2}. \tag{13}$$

The critical stresses σ^0_∞ and σ^*_∞ for cracks of fixed length l_{nk} are the estimates of the beginning and completion of fracture: these critical stresses differ severalfold. Undoubtedly, all the estimates for the critical parameters K^0_{I}, K^*_{I}, l^0_{nk}, l^*_{nk}, σ^0_∞, and σ^*_∞ can be obtained numerically by using the second inequality in (7) and initial relations (8)–(10) (if the solution exists).

Remark 2. When a composite is reinforced by pre-stressed high-strength fibers, other problems arise since the crack in the matrix may not open under low loads.

We estimate the orders of the dimensionless parameters Δ/l^*_{nk}, $\sigma_{m2}/\sigma^*_\infty$, $a\varepsilon_{m2}/l^*_{nk}$, and G/σ^*_∞ in the case where all the assumptions accepted above are valid and relations (12) hold. In the ratio G/σ^*_∞, we pass to Young's modulus E using the relation $G = E/[2(1+\nu)]$; the estimates of the "theoretical" strength in terms of Young's modulus are known: $\sigma_{m1} = (0.1\text{–}0.2)E$ (see the review in [18]). If $a\varepsilon_{m2}/l^*_{nk} \sim 10^{-2}$, then $\Delta/l^*_{nk} \approx 0.06$ for $\sigma_{m2}/\sigma_\infty \sim 10$, $\Delta/l^*_{nk} \approx 0.2$ for $\sigma_{m2}/\sigma_\infty \sim 10^2$, and $\Delta/l^*_{nk} \approx 0.6$ for $\sigma_{m2}/\sigma_\infty \sim 10^3$. Obviously, when $\sigma_{m2}/\sigma_\infty \sim 1$, the critical lengths of the cracks l^0_{nk} and l^*_{nk} can differ by one order of magnitude [see (13)].

Let us estimate the depth of the trap for propagating cracks (see Fig. 5). It is obvious that $2l^*_{nk} = 2l^0_{nk} + 2\Delta$. We confine our consideration to the simple case where $n = k = 1$. Taking into account relations (10) and (11) and ignoring the secondary terms $\Delta/l^0_{11} \ll 1$ and $\sqrt{2l^0_{11}/r_1} \gg 1$, we obtain

$$\frac{\sigma^*_\infty}{\sigma^0_\infty} \simeq \Big(1 - \frac{4\sqrt{2}}{\pi(æ+1)}\,\frac{\sigma_{m2}}{\sigma^*_\infty}\,\frac{a\varepsilon_{m2}}{l^0_{11}}\,\frac{G}{\sigma^*_\infty}\Big)^{-1}.$$

The critical stresses predicted by the necessary and sufficient criteria σ^*_∞ and σ^0_∞ may differ considerably (severalfold) within a certain range of dimensionless parameters that enter the last relations.

The crack resistance of ceramics can be improved [15, 16] by introducing highly deformable additives into their composition.

One can easily find a relation between the critical stresses predicted by the necessary and sufficient criteria proposed by Novozhilov [2] and the local minima and maxima for the crack propagation in Thompson's model [19].

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REFERENCES

1. H. Neuber, *Kerbspannunglehre: Grunglagen für Genaue Spannungsrechnung*, Springer-Verlag (1937).
2. V. V. Novozhilov, "Necessary and sufficient criteria of brittle strength," *Prikl. Mat. Mekh.*, **33**, No. 2, 212–222 (1969).
3. V. M. Kornev, "Hierarchy of strength criteria of structured brittle media. Satellite initiation of microcracks," *J. Appl. Mech. Tech. Phys.*, **41**, No. 2, 367–376 (2000).
4. V. M. Kornev, "Multiscale criteria of shear strength of block brittle media. Satellite initiation of micropores," *Fiz. Tekh. Probl. Razrab. Polezn. Iskop.*, **40**, No. 5, 7–16 (2000).
5. M. Ya. Leonov and V. V. Panasyuk, "Growth of the smallest cracks in solids," *Prikl. Mekh.*, **5**, No. 4, 391–401 (1959).
6. D. S. Dugdale, "Yielding of steel sheets containing slits," *J. Mech. Phys. Solids*, **8**, 100–104 (1960).
7. G. I. Barenblatt, "Mathematical theory of equilibrium cracks formed upon brittle fracture," *Prikl. Mekh. Tekh. Fiz.*, No. 4, 3–56 (1961).
8. I. M. Kershtein, V. D. Klyushnikov, E. V. Lomakin, and S. A. Shesterikov, *Fundamentals of Experimental Fracture Mechanics* [in Russian], Izd. Mosk. Univ., Moscow (1989).
9. S. A. Nazarov and O. R. Polyakova, "Fracture criteria, asymptotic conditions at the crack tips, and self-conjugate expansions of the Lamé operator," *Tr. Mosk. Mat. Obshch.*, **57**, 16–74 (1996).
10. V. M. Kornev and V. D. Kurguzov, "Sufficient discrete–integral criterion of rupture strength," *J. Appl. Mech. Tech. Phys.*, **42**, No. 2, 328–336 (2001).
11. V. M. Kornev and V. V. Adishchev, "Sufficient criteria of growth of normal-rupture cracks in a medium of a regular structure," *Izv. Vyssh. Uchebn. Zaved. Stroit.*, No. 12, 9–14 (1999).
12. N. F. Morozov and N. V. Ponikarov, "Mathematical models in fracture mechanics," in: *Problems of Mechanics of Solids and Structural Elements: To the 60th Anniversary of Prof. G. I. Bykovtsev* [in Russian], Dal'nauka, Vladivostok (1998), pp. 97–104.
13. V. É. Vil'deman, Yu. V. Sokolkin, and A. A. Tashkinov, "Boundary-value problems of continual fracture mechanics," Preprint, Inst. of Continua Mechanics, Ural Div., Russian Acad. of Sci., Perm' (1992).
14. V. V. Panasyuk, A. E. Andreikiv, and V. Z. Parton, "Fundamentals of fracture mechanics," in: *Failure Mechanics and Strength of Materials* [in Russian], Vol. 1, Naukova Dumka, Kiev (1988).

15. D. M. Lipkin, "Effect of interfacial adhesion on plastic dissipation in metal–ceramic fracture," in: *Fifth Int. Conf. on the Fundamentals of Fracture* (ICFF-V) (Gaithersburg, Aug. 18–21, 1997), Report No. 97-13, Inst. for Mech. and Mater., La Jolla (1997), p. 135.
16. Q. Ma, "Subcritical crack growth along interfaces in interconnect structures," *ibid.*, p. 139.
17. M. P. Savruk, "Stress-intensity coefficients for cracked bodies," in: *Failure Mechanics and Strength of Materials* [in Russian], Vol. 2, Naukova Dumka, Kiev (1988).
18. N. H. Macmillan, "The ideal strength of solids," in: R. Latanision and J. R. Pickens (eds.), *Atomistics of Fracture*, Plenum Press, New York (1983), pp. 95–164.
19. R. Thompson, "Physics of fracture," *ibid.*, pp. 167–204.